Relational Mechanics and the Diurnal Rotation of the Earth

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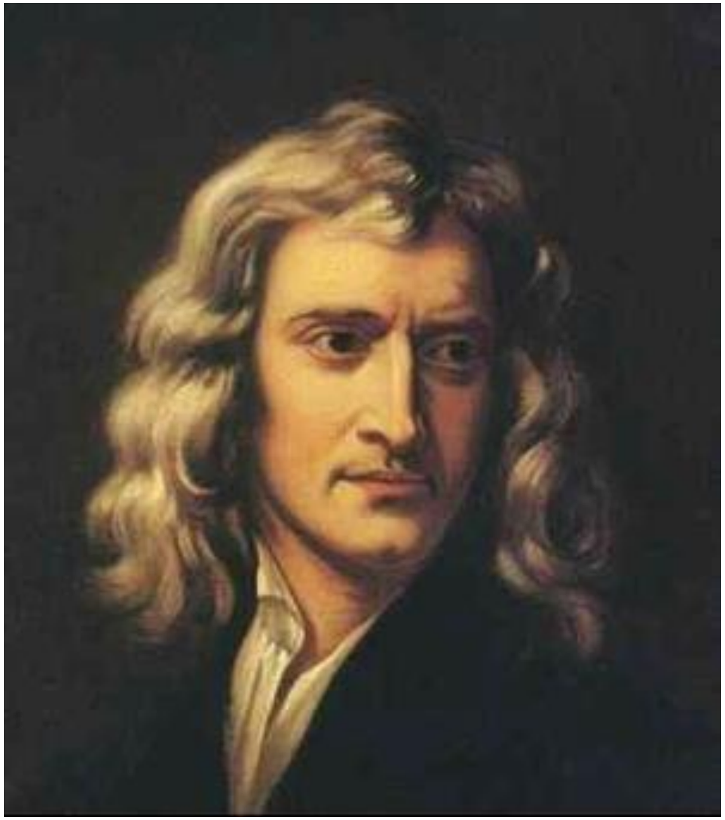
Isaac Newton (1642 – 1727)

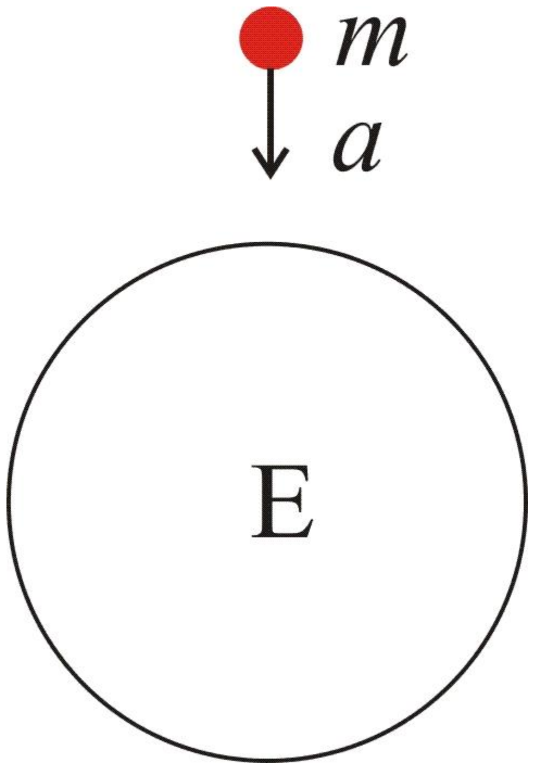


1687: Principia

$$F = ma$$

$$F = G\frac{m_1 m_2}{r^2}$$

Free fall in Newtonian mechanics

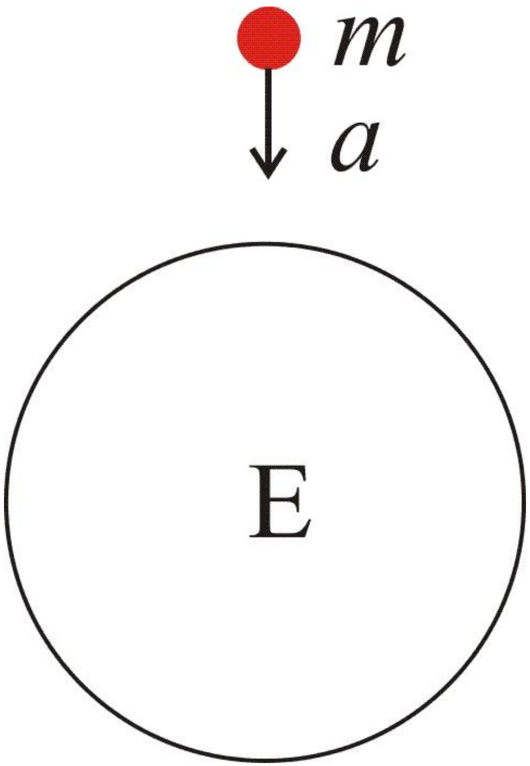


$$F = ma$$

$$mg = ma$$

$$a = g = \frac{GM}{R^2} = 9.8\,\frac{m}{s^2}$$

Free fall in Newtonian mechanics



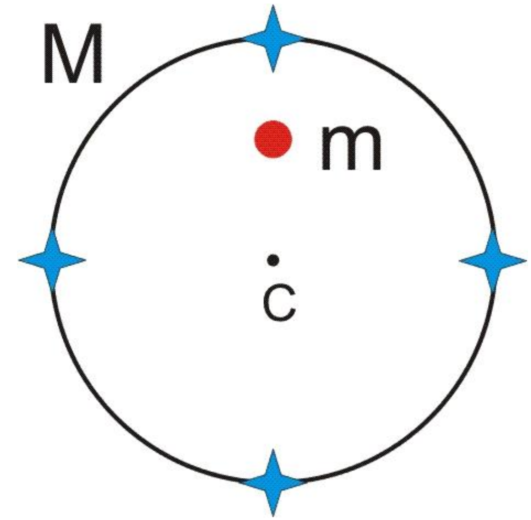
$$F = ma$$

$$mg = ma$$

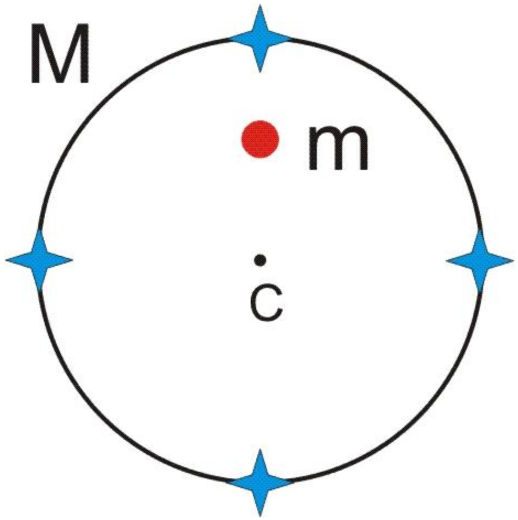
$$a = g = \frac{GM}{R^2} = 9.8\,\frac{m}{s^2}$$

However, this is <u>not</u> a two-body problem. There are also stars and galaxies around the Earth.

$$F = G\frac{m_1 m_2}{r^2}$$



$$F = G\frac{m_1 m_2}{r^2}$$

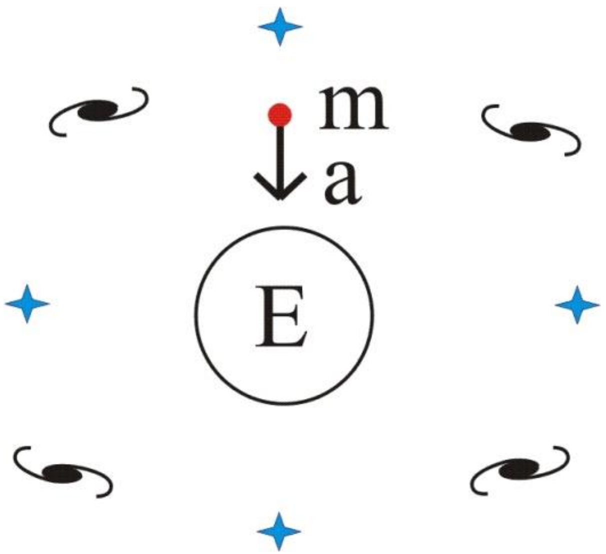


Newton in the Principia:

"Theorem 30: If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from these points, I say, that a particle placed within that surface will not be attracted by these forces any way."

$$F = 0$$

Correct way to solve the problem of free fall in Newtonian mechanics:



$$F_E + F_* = ma$$

$$G\frac{mM_E}{R_E^2} + 0 = ma$$

$$a = \frac{GM_E}{R_E^2} = 9.8\, m/s^2$$

Newton in the Principia:

"Absolute space, without relation to anything external, remains always similar and immovable."

Newton's 2nd law of motion:

$$F = ma$$

The free fall acceleration of 9.8 m/s^2 is the acceleration of the apple relative to empty space or relative to the vacuum.

How can we know that the Earth really rotates once a day around its axis?

Cannot all phenomena be also explained supposing the set of stars and galaxies rotating together once a day around the axis of a stationary Earth?

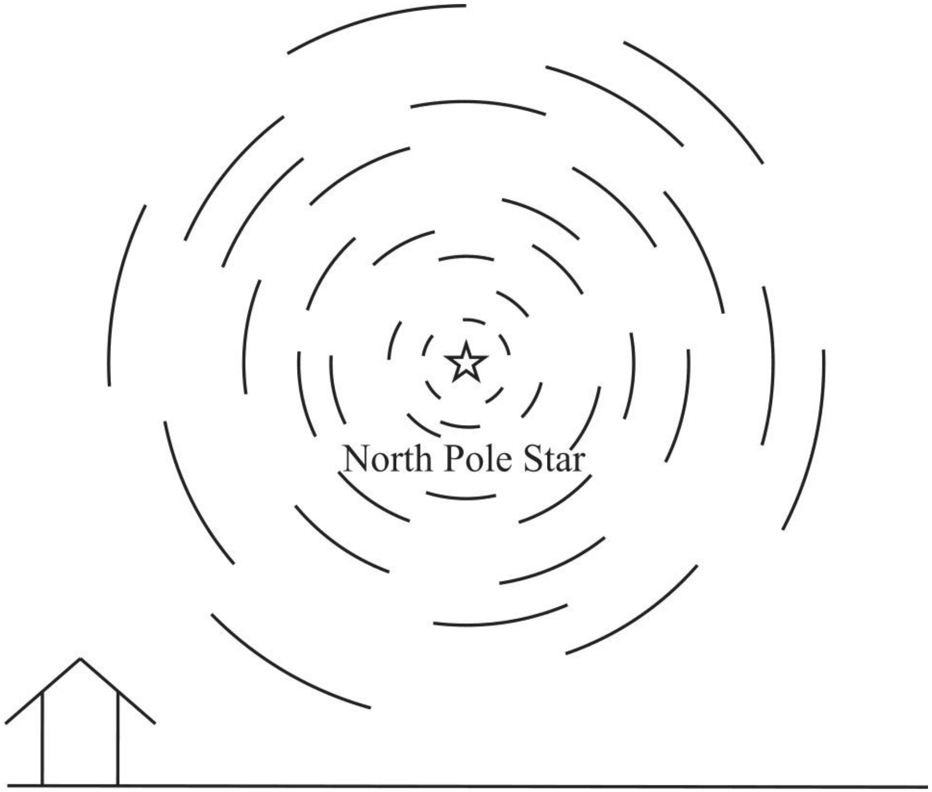
How can we distinguish empirically the Ptolemaic and the Copernican world views?

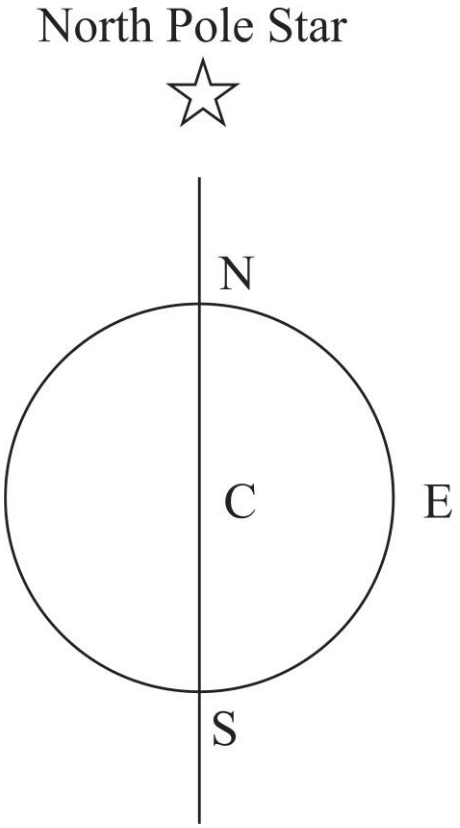
There are two kinds of diurnal rotations of the Earth:

a) The kinematic rotation which is measured by the relative rotation between the Earth and external bodies (stars and galaxies).

b) The absolute rotation which is measured by dynamic effects (the flattening of the Earth and Foucault's pendulum).

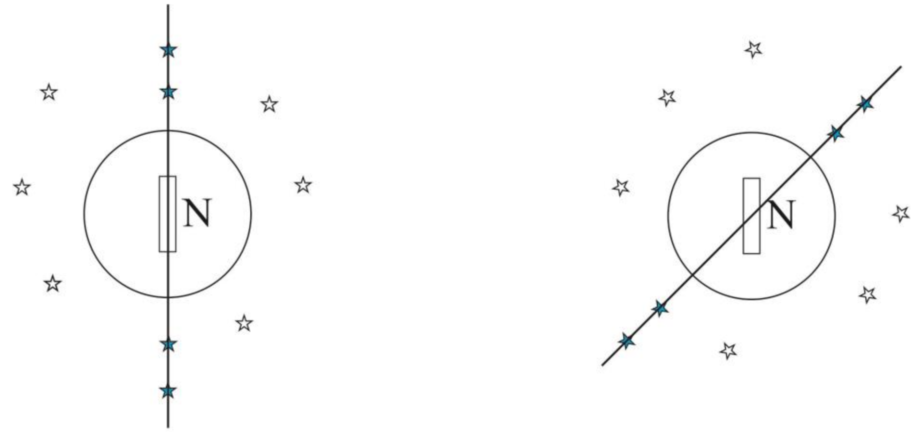
The kinematic or relative rotation of the Earth:





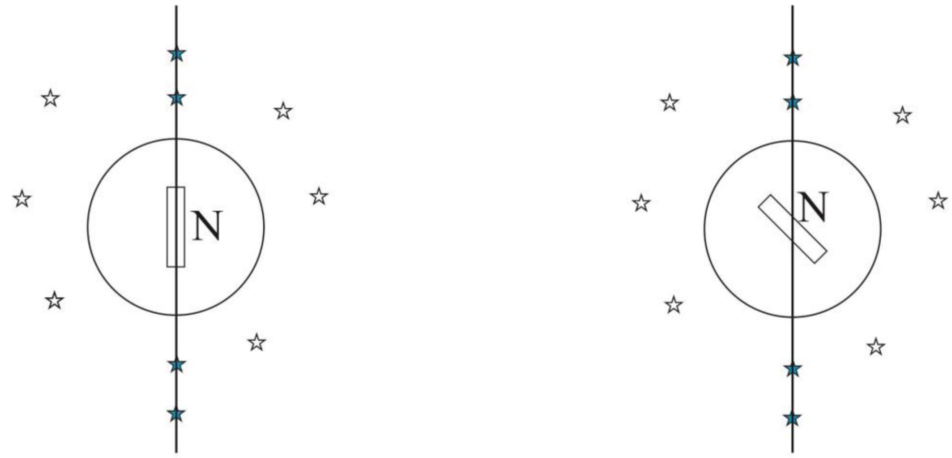
Period = T_R = 24 hours

This motion can be interpreted as the stars rotating clockwise around a fixed Earth:



(a) (b)

Or equivalently as the Earth rotating anti-clockwise around the fixed stars:

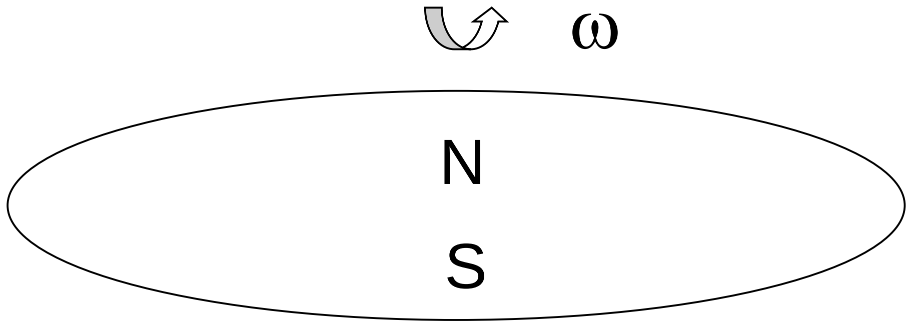


(a) (b)

The Ptolemaic and Copernican world views are kinematically equivalent.

We now discuss another kind of rotation of the Earth, namely, the absolute or dynamic rotation. In principle it has no relation with the stars and galaxies around the Earth. It can be detected and measured without looking at the night sky.

The absolute rotation of the Earth as measured by its flattening at the poles:

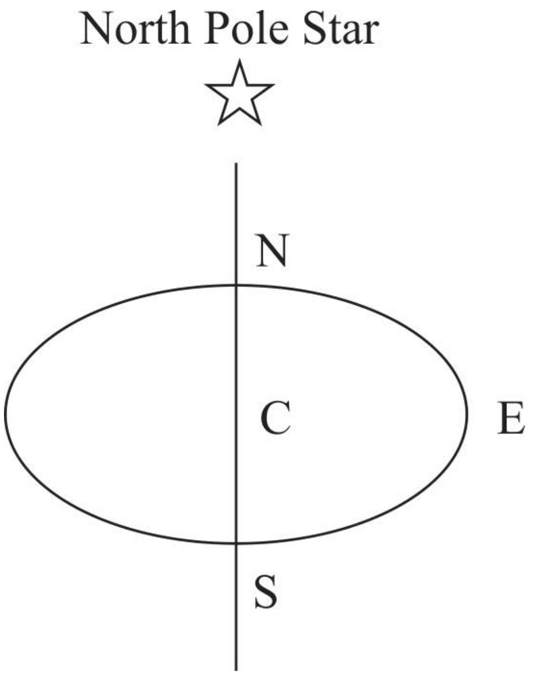


Newton: “The diameter of the earth at the equator is to its diameter from pole to pole as 230 to 229.”

$$\frac{D_E}{D_P} = 1 + \frac{15}{16\pi\, G}\,\frac{\omega_{EAS}^{2}}{\rho_{\mathrm{E}}} = \frac{230}{229} = 1.004$$

“EAS” here means rotation of the Earth relative to Absolute Space.

Two coincidences:



a) The axis obtained by the absolute rotation of the Earth (defined by its flattening) coincides with the axis obtained by the relative rotation between the Earth and the set of stars.

b) In order to obtain the observed flattening of the Earth of 1.004, the absolute period of rotation must be equal to its relative period of rotation relative to the stars:

$$T\_absolute = T\_relative = 24 \text{ hours}$$

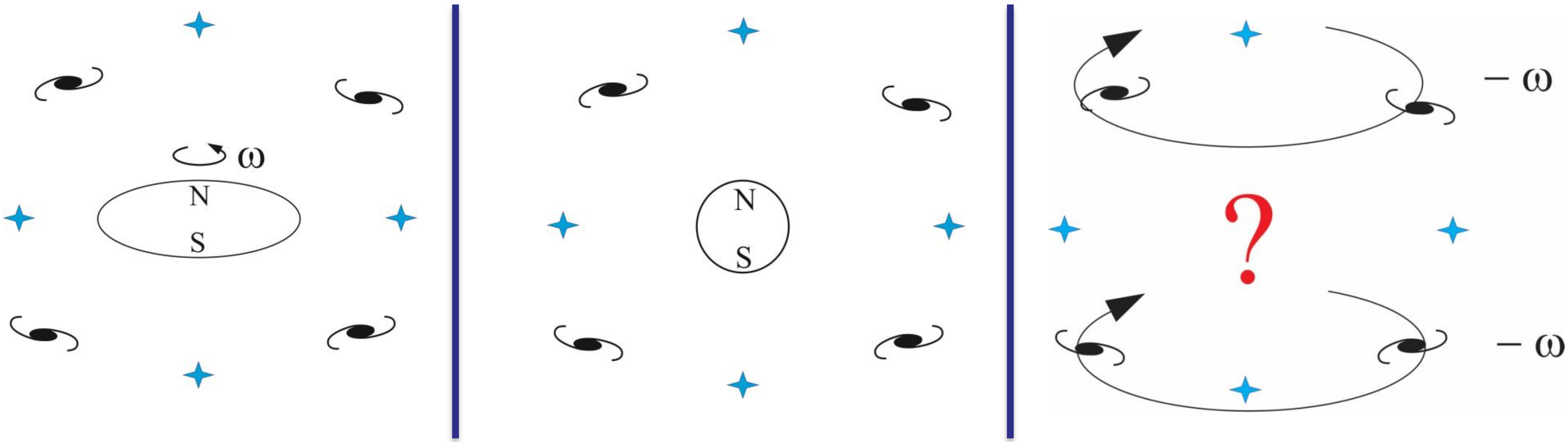
Ernst Mach in The Science of Mechanics, 1883:

“The principles of mechanics can be so conceived, that even for relative rotations centrifugal forces arise.”

He also had the following idea:

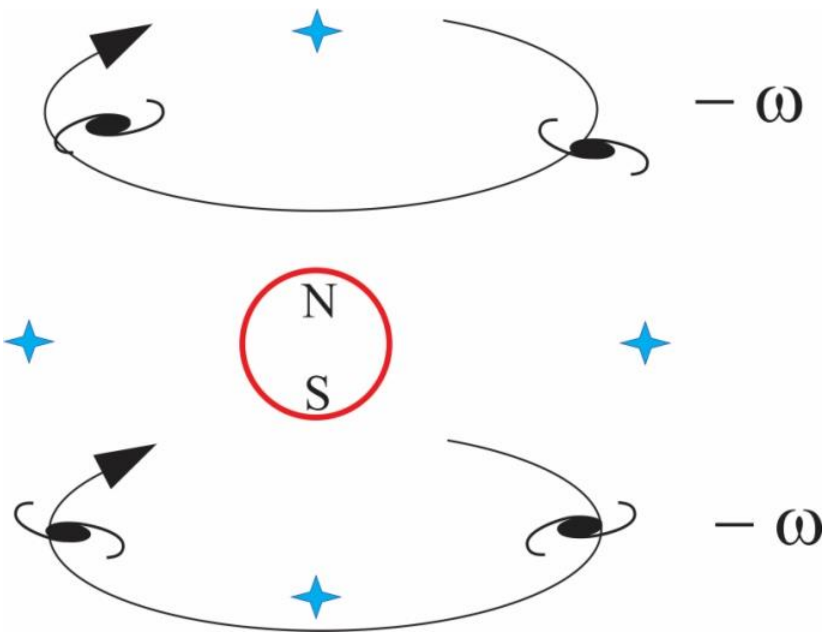
Try to stop the Earth and rotate the heaven of fixed stars, and then prove the absence of centrifugal forces.

Shape of the Earth



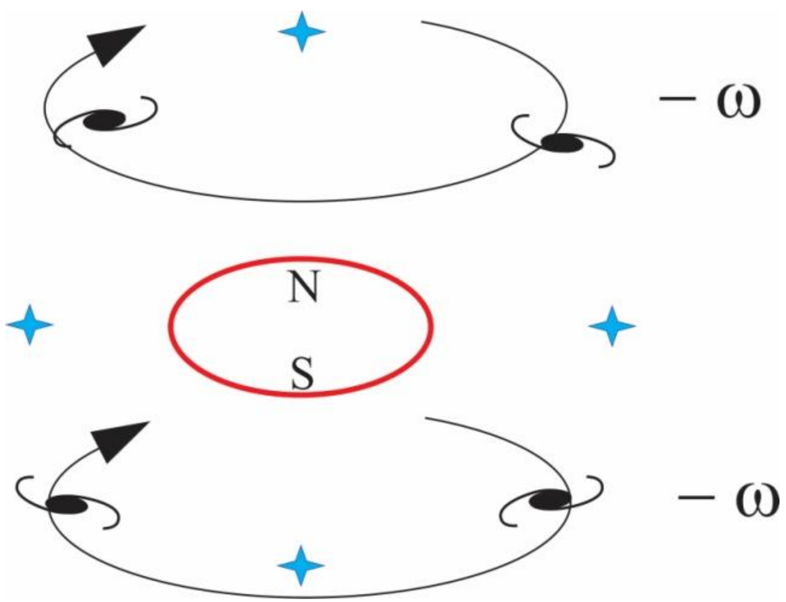
What would be the shape of the Earth if it remained at rest, and the set of stars and galaxies rotated together around its North-South axis in the opposite direction with a period of 24 hours?

Newton



Mach,

Relat. Mech.



Einstein, The Meaning of Relativity, 1922:

"What is to be expected along the line of Mach's thought? A rotating hollow body must generate inside of itself a Coriolis field, and a radial centrifugal field as well."

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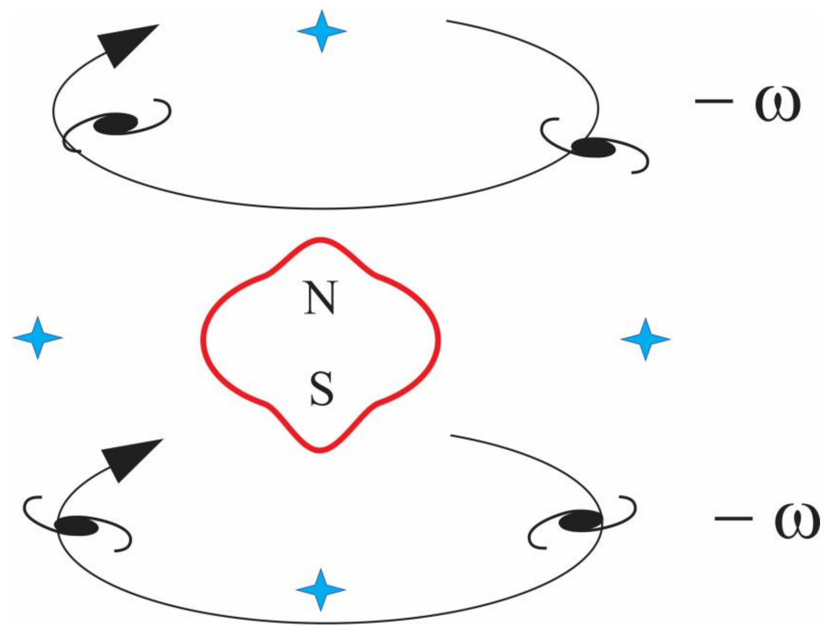
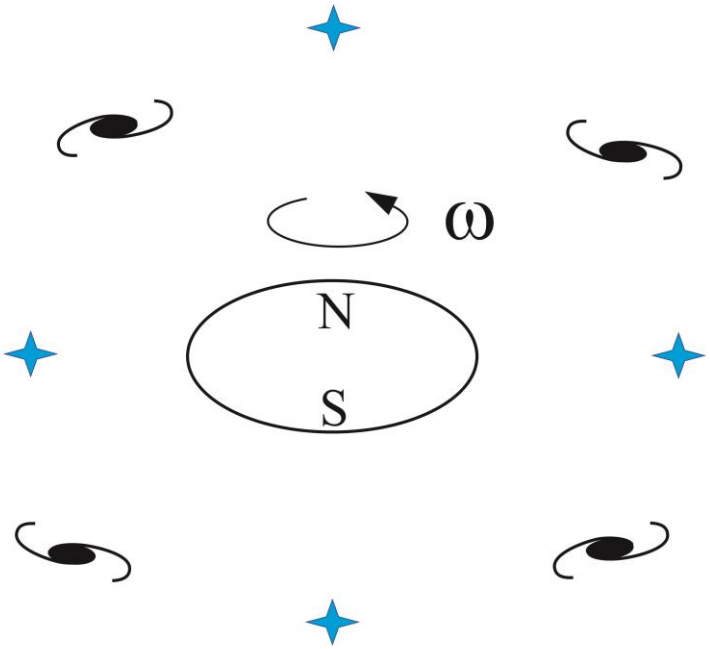
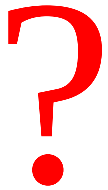
However, according to general relativity (Lense-Thirring effect):



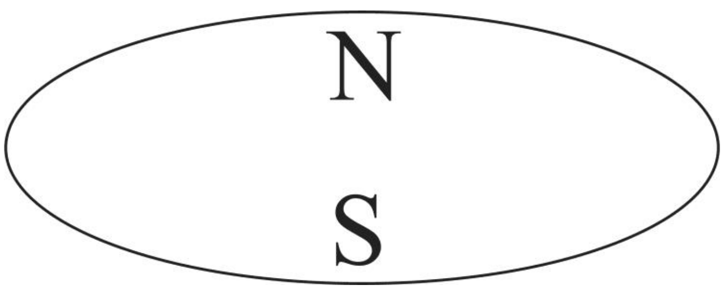
Carl Neumann (1869): “What would be the shape of the earth if all other astronomical bodies were annihilated?”





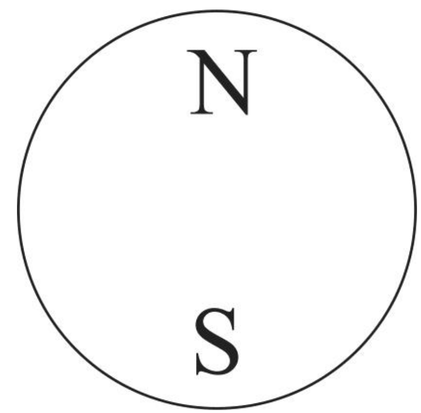
Newton,

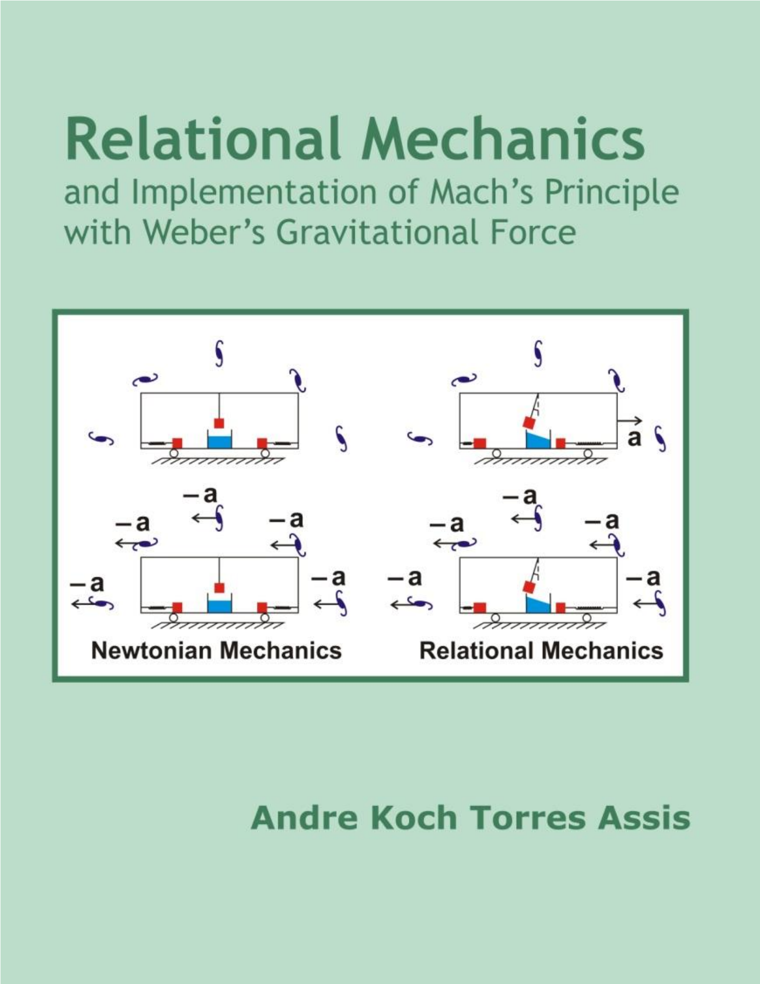
Einstein



Mach,

Relat. Mech.





Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force

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Postulates of Relational Mechanics:

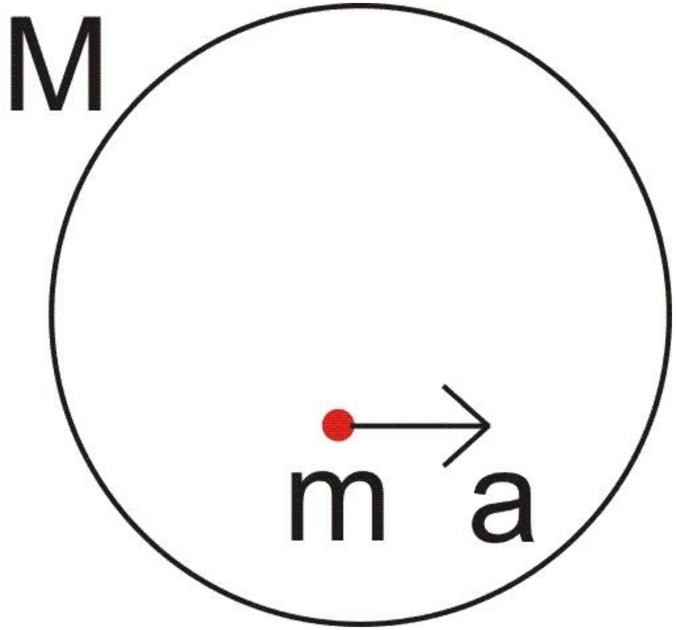
The sum of all forces acting on any body is always zero in all frames of reference.

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Main calculation of Relational Mechanics:

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Force exerted by a stationary spherical shell of mass *M* acting on a test body *m* which is moving with acceleration *a* inside it:



$$\vec{F}_{\text{Newton}} = \vec{0}$$

Main calculation of Relational Mechanics:

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

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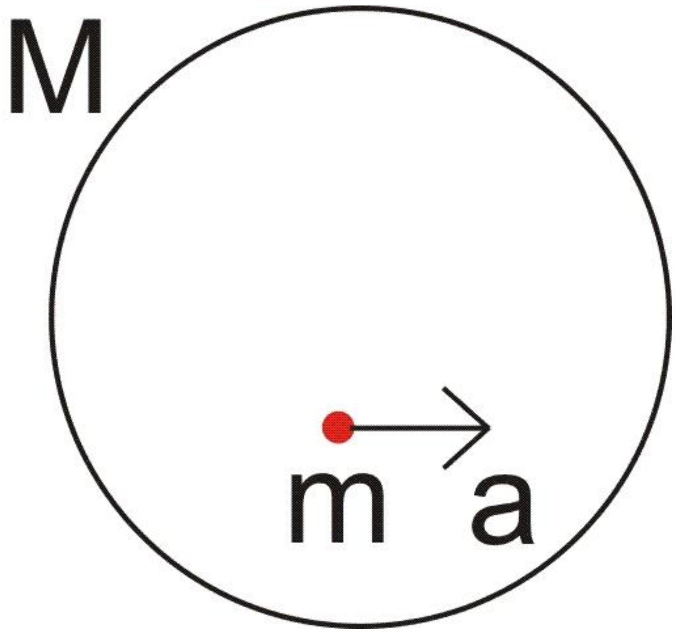


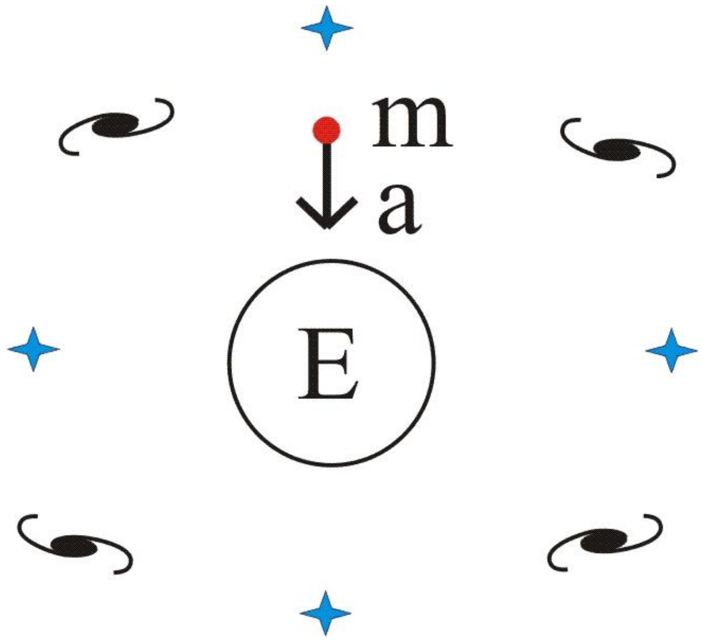
$$\vec{F}_{\text{Newton}} = \vec{0}$$

$$\vec{F}_{\text{Relat. Mech.}} = -\phi\, m_g \vec{a}$$

with $\phi = \frac{2H_g M_g}{Rc^2}$

Free fall in Relational Mechanics:

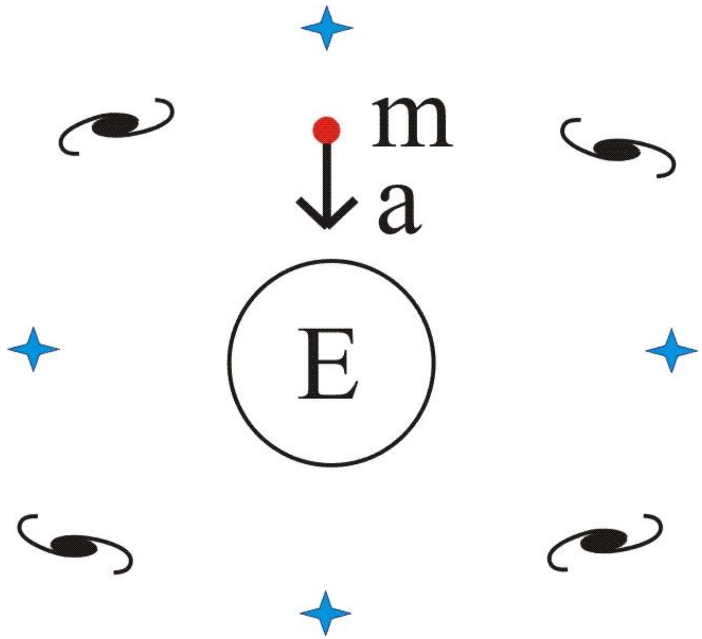
$$F_E + F_* = 0$$



Free fall in Relational Mechanics:

$$F_E + F_* = 0$$

$$H_g \frac{m_g M_{gE}}{r^2} - \Phi m_g a = 0$$



$$\boxed{a = \frac{H_g}{\Phi} \frac{M_{gE}}{r^2}}$$

with $$\frac{H_g}{\Phi} = \frac{H_o^2}{4\pi \rho_*} \approx 6.7 \times 10^{-11} \frac{Nm^2}{kg^2} = G$$

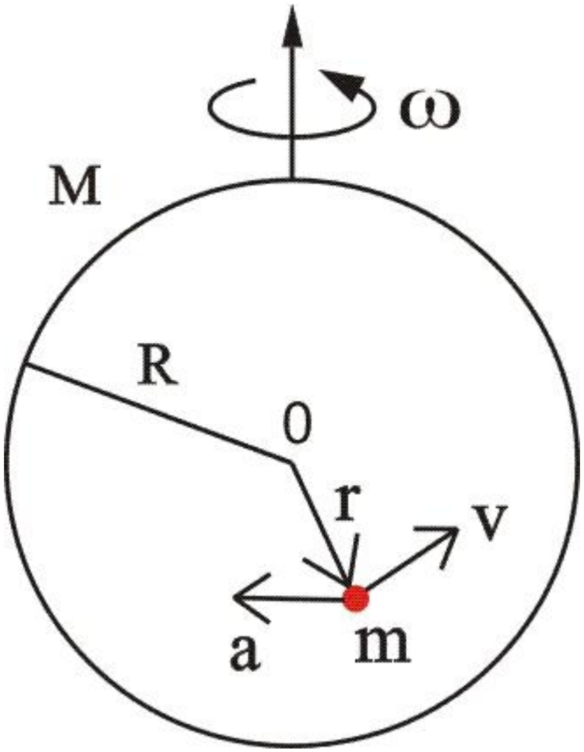
The previous slide presents the essence of Relational Mechanics.

We begin with the postulate that the sum of all forces acting on any body is always zero. Then we deduce an expression analogous to Newton's second law of motion, namely:

$$\vec{F} - m_g \vec{a}_* = \vec{0}$$

In this equation F represents the usual forces acting on the test body. The expression $-ma$ represents the real gravitational force exerted by the set of distant galaxies acting on the test body, according to Weber's law applied to gravitation. The mass appearing here is the gravitational mass of the test body (that is, it is not the inertial mass, as in Newtonian mechanics). The acceleration appearing here is the acceleration of the test body relative to the frame of distant galaxies (that is, it is not the acceleration of the test body relative to absolute space, as in Newtonian mechanics).

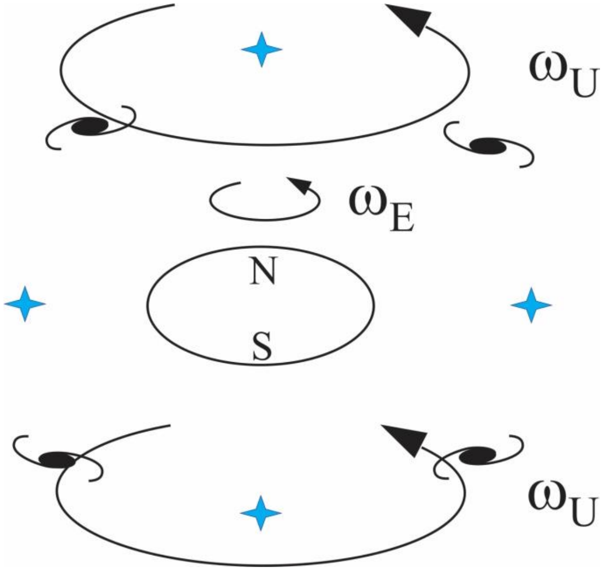
Weber's gravitational force exerted by a spinning shell acting on a particle moving inside it:



$$\vec{F} = -\frac{2H_g M_g}{Rc^2} m_g \left(\vec{a} + \vec{\omega} \times \vec{\omega} \times \vec{r} + 2\vec{v} \times \vec{\omega} \right)$$

Weber's force has a <u>real</u> centrifugal component and a <u>real</u> Coriolis component.

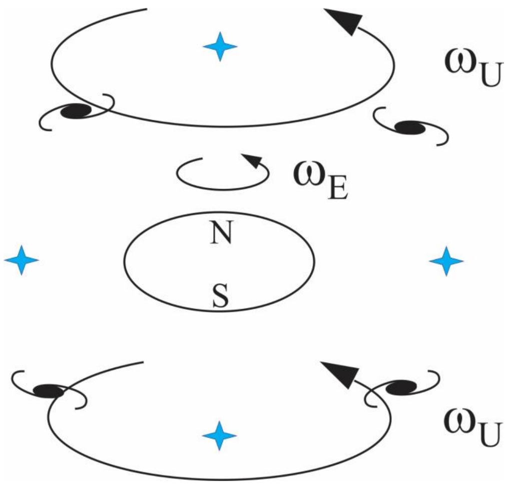
Flattening of the Earth



$$\frac{D_E}{D_P} = 1 + \frac{15}{16\pi G} \frac{\omega_{EAS}^2}{\rho_E} = \frac{230}{229} = 1.004$$

In Newtonian mechanics, the flattening of the Earth <u>does not depend</u> on the rotation of the set of stars and galaxies around the Earth.

Flattening of the Earth



According to Newton:

$$\frac{D_E}{D_P} = 1 + \frac{15}{16\pi G}\frac{\omega_{EAS}^2}{\rho_E} = \frac{230}{229} = 1.004$$

According to Relational Mechanics:

$$\frac{D_E}{D_P} = 1 + 4\frac{\rho_*}{\rho_E}\frac{(\omega_E - \omega_{Universe})^2}{H_o^2} = \frac{230}{229} = 1.004$$

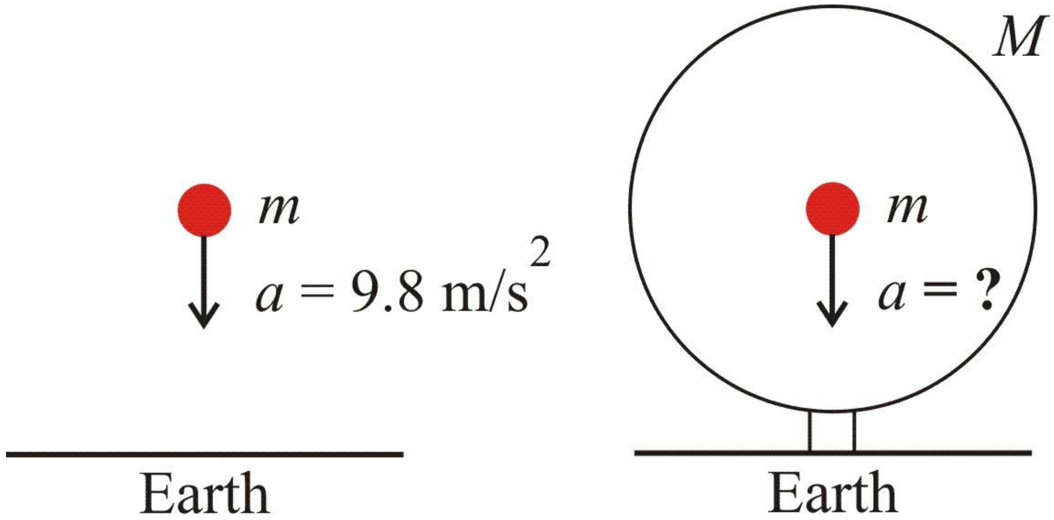
Relational Mechanics leads to two main consequences:

1) The flattening of the Earth depends on the RELATIVE rotation between the Earth and the set of distant galaxies. The Earth will be flattened not only in the Copernican world view (the Earth spinning daily relative to a background of stationary stars and galaxies), but also in the Ptolemaic world view (a stationary Earth surrounded by a set of stars and galaxies spinning together once a day around the Earth's North-South axis).

2) Moreover, the numerical value of the flattening depends now on the amount of external bodies around the Earth. If we could make the average density of mass in the universe go to zero (for instance, by annihilating all stars and galaxies), then the Earth would become spherical.

We can answer Carl Neumann's 1869 question quantitatively as follows: The shape of the Earth would be spherical, if we could annihilate all other astronomical bodies!

Experimental test 1: What will be the acceleration of free fall when the test body is surrounded by a stationary spherical shell of mass M?



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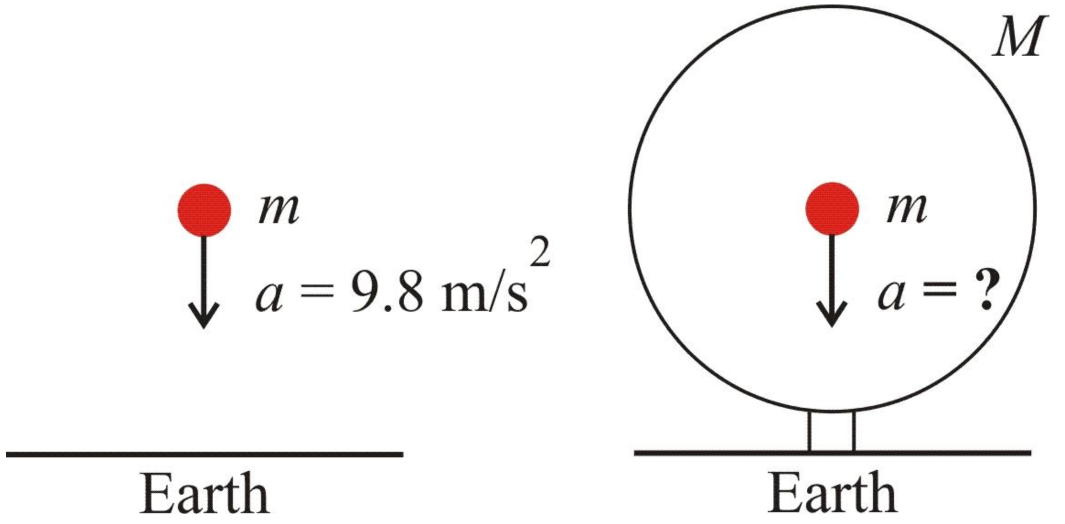


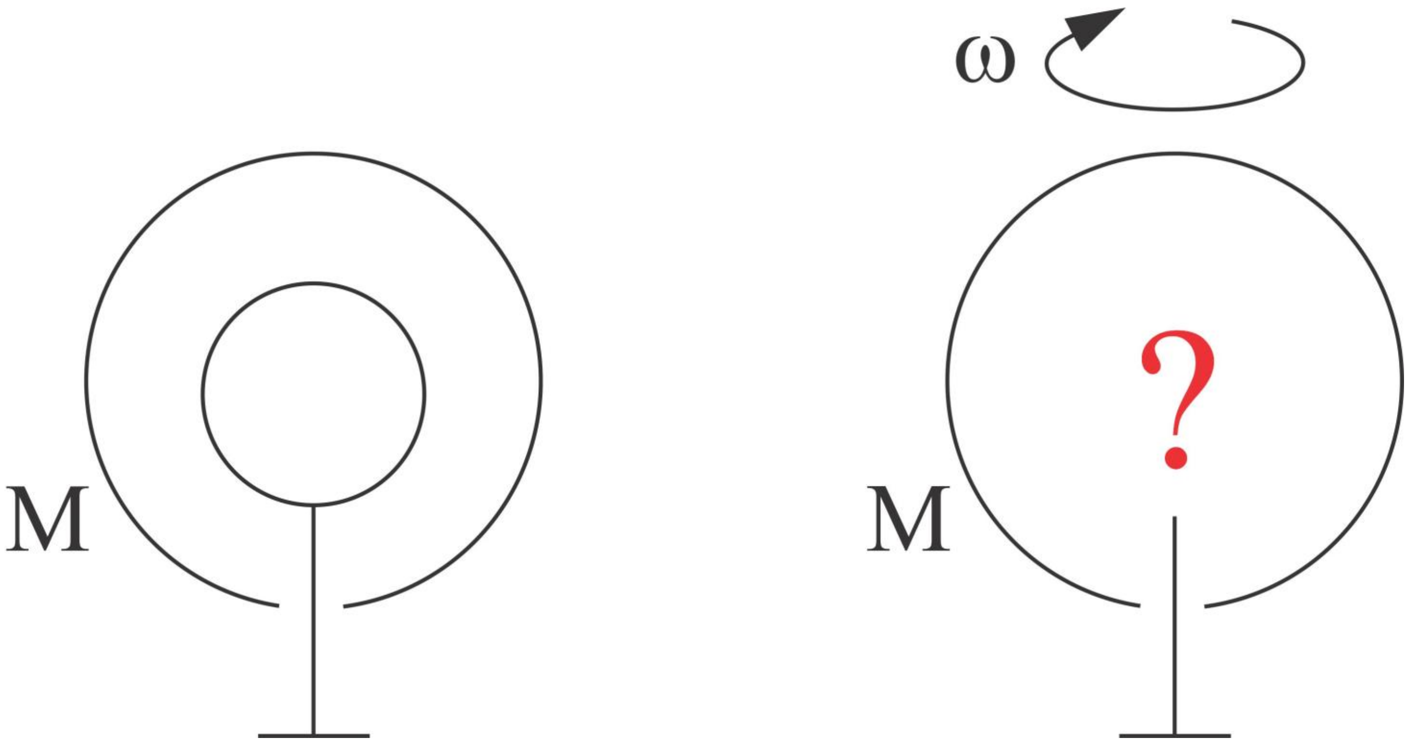
$$a_{Newton} = a_{Einstein} = g = 9.8\,m/s^2$$

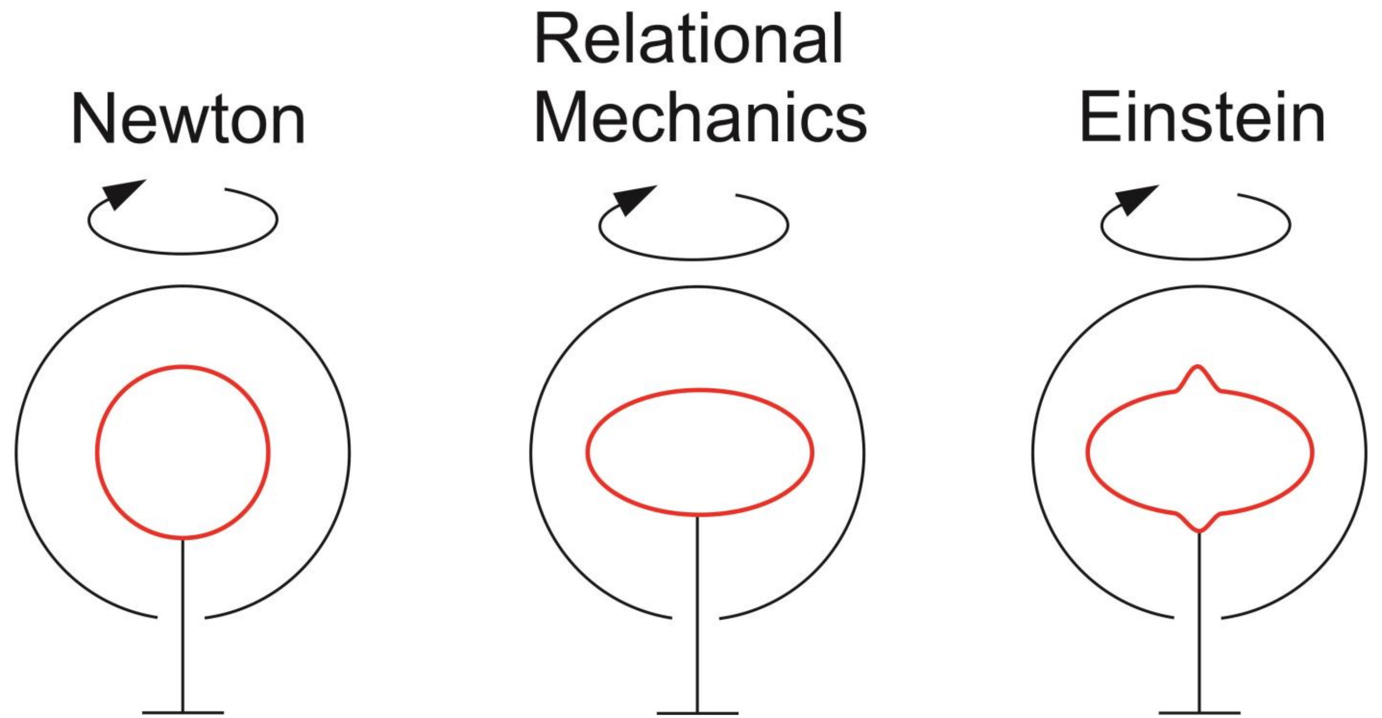
$$a_{\text{Relat. Mech.}} = g\left(1 - \frac{2GM}{Rc^2}\right)$$

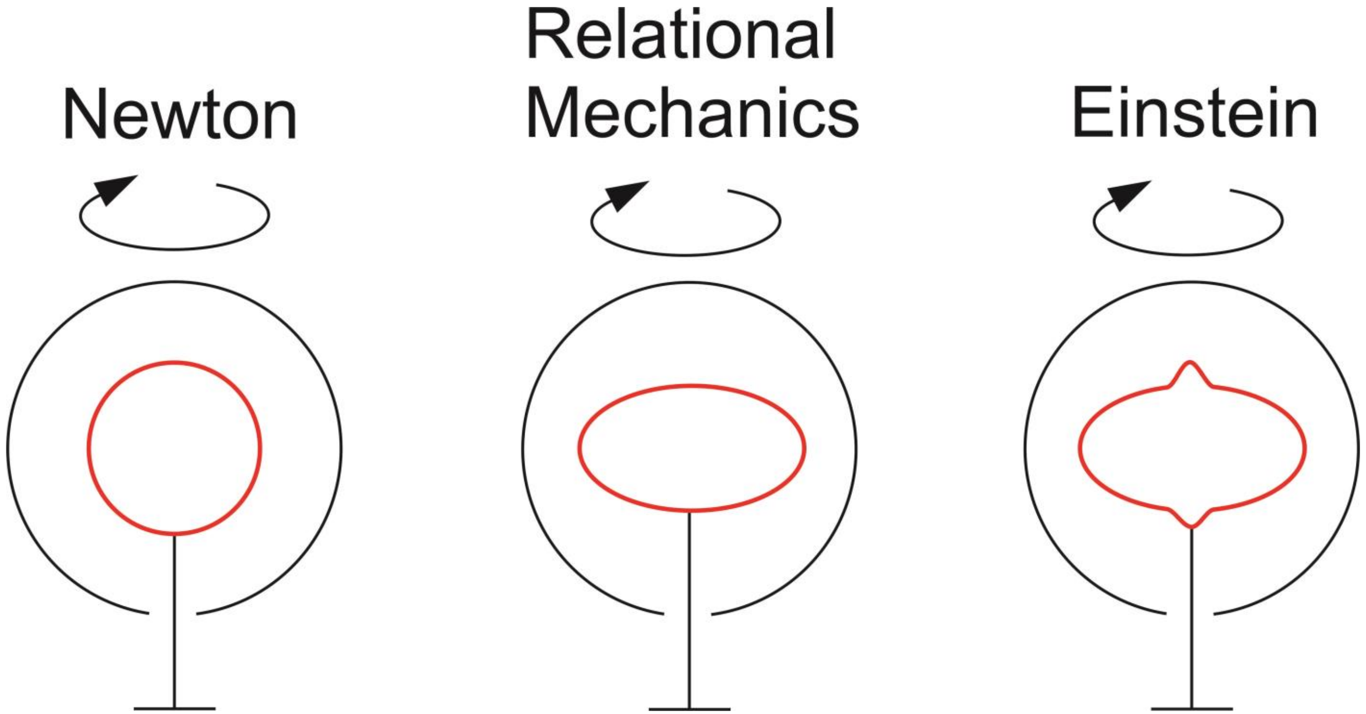
order of magnitude:

If $R = 1m$ and $M = 10^3 kg$ then $\dfrac{2GM}{Rc^2} = 10^{-24}$

Experimental test 2: Consider an elastic sphere at rest relative to the ground. We place a spherical shell of mass M around the sphere. What will be the shape of the internal sphere if only the external shell rotates uniformly relative to the ground around the axis of the bucket?




Newton
Relational
Mechanics
Einstein



Unfortunately we cannot distinguish in practice these 3 different predictions because the amount of flattening or deformation is so small that all 3 shapes of the internal elastic sphere will be essential spherical (supposing, for instance, a spherical shell of radius 1 meter, a mass of 1000 kg, spinning relative to the ground with 1 radian per second).

In any event, it is important to see these different predictions. In principle these predictions may be distinguished experimentally in the future.

Conclusion

- Relational Mechanics implements quantitatively Mach's principle.
- The flattening of the Earth and Foucault's pendulum are due to a real gravitational interaction with the set of distant galaxies.
- These phenomena take place not with the Earth spinning once a day relative to a stationary set of galaxies, but also with the set of galaxies rotating daily around the NS axis of a stationary Earth.
- The Ptolemaic and Copernican world views are now equivalent not only kinematically, but also dynamically.

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