Symplectic Topology meets Celestial and Quantum Mechanics via Weber* Electrodynamics

Mathematics meets Physics

Advanced School
UNICAMP, Campinas, Brazil
17 – 21 February 2020

Organizer: Joa Weber

February 26, 2020

Abstract

Contents

*Wilhelm Eduard Weber 1804 – 1891

1 Aim of the Summer School

The aim is to bring together the fields of Symplectic Topology (Mathematics) and Celestial Mechanics (Mathematics/Physics), motivation coming from the recent research field of

non-local Floer homology

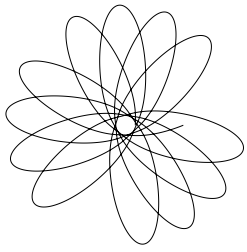
(Mathematics) and a 150 year old example from (the History of Physics) called Weber Electrodynamics (WED), in order to start a long-term interaction.
We will have five **introductory** mini-courses, whose character is to present overview and ideas, four by mathematicians, international leaders in symplectic topology (incl. celestial mechanics) or Lorentzian geometry, and one on Weber Electrodynamics by one of the world's leading experts on the history of electrodynamics. Moreover, we will have one or two further talks on delay equations by internationally renowned experts.

As the meeting is at the beginning of a research field, non-local Floer homology, it is an **ideal research entrance point** for graduate students. Furthermore, there is room for plenty of discussion in order to foster long-term collaborations and to enhance the intercontinental research contacts between Brazil and Germany. The meeting is meant to be an **advanced school** not only to graduate students and post-docs, but also to the lecturers themselves.

We hope some lecturers can stay for a few days after the school, so that the inputs from the other fields courses can be discussed (without having to prepare next days talk) and joint research projects can be established.

Characteristics

- Interdisciplinarity: Mathematics - Physics
- International: Brasil - Germany
- Extraordinary high profile lecturers
- Great opportunity for graduate students to enter research



2 Background

The **construction of non-local Floer homology** is currently a topic of intense research [AFS19b, AFS18, AFS19a]. It is motivated by the discovery of new smooth structures[1] in infinite dimensions by Hofer, Wysocki, and Zehnder [HWZ17]. The new point in non-local Floer homology is that it is assigned to

delayed differential equations.

An example of which is the (almost) forgotten dynamics associated to the Hamiltonian introduced by Wilhelm Weber [Web46] in 1846 and, most importantly (cf. [Fra19]), generated by a retarded potential functional as observed by von Neumann [Neu68] back in 1868.

Different from Maxwell's field theory Weber's electrodynamics is an action-at-a-distance theory and therefore leads to a Hamiltonian system on a finite dimensional space. The Hamiltonian in Weber's electrodynamics has interesting relations with Riemannian geometry, Lorentzian geometry, but as well to Hamiltonian delay equations. Moreover, Weber's electrodynamics has also interesting connections to celestial mechanics, in particular, to Mach's principle; see [Ass14]. Among the most surprising features of Weber's theory is that it predicts a **critical radius** inside which reigns Lorentz geometry and equal charge particles (such as in an atomic nucleus) attract each other, whereas outside reigns Riemannian geometry and they are repulsive; see [FW19].

Unfortunately, Weber's electrodynamics is barely known in the symplectic community and its big potential as a driving force in the development of new techniques in symplectic topology has so far not been taken advantage of. It is a lucky coincidence that André Koch Torres Assis, one of the world's leading experts on the history of Weber's electrodynamics [Ass94], and myself are working at the same university in Brazil, namely UNICAMP in Campinas. During the visit of Urs Frauenfelder from February 5-13 2019 at UNICAMP the three of us had intense fruitful discussions which convinced us about the great potential an interdisciplinary Workshop on this topic might have. An outgrowth of these discussions is a paper in which we pioneered the symplectic point of view of Weber's electrodynamics. The relevance of the interaction was recognized by a rather nice Journal which immediately published our paper [FW19].

In view of this, and, in particular, thanks to the relation of Weber's Hamiltonian with the recently introduced class of **Hamiltonian delay equations** [AFS18] we expect that Weber's force law, more precisely its von Neumann potential [Fra19], will become a key example and hopefully a source for ideas in the **construction of non-local Floer homologies**.

[1] On this recent theory I gave a lecture course at UNICAMP in 2018, writing Lecture Notes available in the AMS Open Math Notes project [Web18], and also an advanced mini-course during the Colóquio Brasileiro de Matemática CBM-32 in August 2019 at IMPA.

3 Program

The character of this meeting is not the one of a conference with many talks. Instead we wish to provide an **initial step** in the construction of non-local Floer homologies and, in particular, study how the historic example of *Weber's Planetary Model of the Atom* fits into this and relates to *Symplectic Topology*. From our mathematical point of view the school focuses on

non-local Floer homologies and **Hamiltonian delay equations**.

The mini-courses are carefully selected with this in mind and we wish to leave time for discussion and people to make contact.

The following mini-courses Cx (3-5 lectures each) are **confirmed**:

C1 Maths: 5 lectures **"Symplectic Aspects of Celestial Mechanics"**
(2x Inventiones, h17, 913/48=19 citations/papers) Kai Cieliebak (Augsburg)
(Editor Inventiones mathematicae)

C2 Physics: 5 lectures **"Weber's Electrodynamics & Atom Model"**
(37 books and 120 papers) André Assis (IFGW UNICAMP)

C3 Maths: 5 lectures **"Non-local Floer Homology"**
(h14, 593/55=11 cits./papers) Urs Frauenfelder (Augsburg)

C4 Maths: 3 lectures **"Celestial mechanics and systolic geometry"**
(2x Inventiones) Pedro Salomão (IME USP)

C5 Maths: 3 lectures **"Introduction to Lorentzian geometry"**
Stefan Suhr (Bochum)

The following speakers for additional talks are **confirmed**:

T1 Talk Maths: **"t.b.a."**
(h11, 425/40=11 cits./papers) Clodoaldo Grotta Ragazzo (IME USP)

For a first/quick glance at some topics: (non-exhaustive list)

- Celestial Mechanics / Hamiltonian dynamics [Gei16, FvK18]
- Symplectic geometry / Floer homology [Web17, AS19]
- Hamiltonian delay equations [AFS18, AFS19b]
- Weber's theory of electrodynamics WED: Our recent paper [FW19] recollects a few details of the somehow unfortunate history of WED. Moreover, André's homepage contains a lot of literature, to start with we recommend
  - Weber's Electrodynamics (Assis, André)
  - Weber's Planeten-Modell des Atoms (Wolfschmidt, Gudrun and Koch Torres Assis, André and Wiederkehr, Karl Heinrich)
  - O Modelo Planetário de Weber para o Átomo (Wolfschmidt, Gudrun and Koch Torres Assis, André and Wiederkehr, Karl Heinrich)

3.1 Courses

3.1.1 C1 - Cieliebak - Symplectic Aspects of Celestial Mechanics

Abstract. This lecture series presents an introduction to celestial mechanics, focusing on its symplectic aspects and some recent advances. In particular, it will cover the following topics: integrability versus chaotic behaviour, regularization of 2-body collisions, the theory of global surfaces of section, and new invariants for families of periodic orbits in the planar circular restricted 3-body system. References [CFvK17, CFZ19]. There will be five lectures:

1. **The N-body problem**
   - Newtonian and Hamiltonian formulation
   - Conserved quantities
   - Kepler problem and its regularizations
   - Non-integrability of the 3-body problem
2. **Chaotic Behavior**
   - Restricted 3-body problem
   - Sitnikov's system and symbolic dynamics
   - Poincare sections
   - Homoclinic orbits and Smale's horseshoe
3. **Global surfaces of section**
   - Definition and examples
   - Birkhoff's conjecture
   - Energy hypersurfaces of contact type
   - Theorem of Hofer-Wysocki-Zehnder
   - Convexity and dynamical convexity
4. **Stark-Zeeman systems**
   - Planar circular restricted 3-body problem and its limit cases
   - Stark-Zeeman systems
   - Periodic orbits and plane curves
   - Families of periodic orbits and Stark-Zeeman homotopies
5. **J^+ type invariants**
   - Arnold's invariant J^+
   - Invariants under Stark-Zeeman homotopies
   - Computations for the Euler and rotating Kepler problem
   - Extension to 2-center Stark-Zeeman systems

3.1.2 C2 - Assis - Weber's Electrodynamics, Relational Mechanics, and Weber's Planetary Model of the Atom

1. **Ampère's Electrodynamics**
   - Oersted's Experiment and Its Interpretation
   - Ampère's Deduction of His Force between Current Elements
   - Ampère's Unification of Magnetism, Electromagnetism and Electrodynamics
   - Modern Experiments to Distinguish Ampère's Force from Grassmann-Biot-Savart's Force
2. **Weber's Electrodynamics**
   - Weber's Force and Potential Energy
   - Deduction of the Laws of Gauss, Ampère and Faraday
   - The Meaning of the Velocity in Lorentz' Force and the Several Definitions of the Field Concept
   - Modern Experiments to Distinguish Weber's Force from Lorentz' Force
3. **The Electric Force of a Current**
   - Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents
   - The Force Between a Closed Circuit Carrying a Constant Current and an External Charge at Rest Relative to the Resistive Circuit
   - Potential and External Electric Field in Circuits with Different Geometries: Straight Wire, Coaxial Cable, Twin-Lead, Solenoid, Ring
   - Propagation of Electromagnetic Signals at Light Velocity from Weber's Law
4. **Weber's Planetary Model of the Atom**
   - Evolution of Weber's Model of the Atom
   - The Two-Body Problem in Weber's Electrodynamics
   - The Critical Distance and the Possibility of a Negative Effective Inertial Mass in Weber's Law
   - Weber's Model of the Atom and the Stability of the Nucleus with his Electrodynamic Force
5. **Relational Mechanics**
   - Newtonian Mechanics: Absolute Space and the Free Fall of an Apple
   - The Kinematic Rotation of the Earth Relative to the Stars and Galaxies

– The Dynamic Rotation of the Earth as Measured by Its Flattening at the Poles and by Foucault's Pendulum
– Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force
– The Rotation of the Earth According to Newton, Mach, Einstein and Relational Mechanics

Bibliography: [Ass94], [AH07], [WAW11], [Ass14], [AC15] and [WAW18].
Some references are available on this homepage.

3.1.3 C3 - Frauenfelder - Nonlocal Floer homology

Abstract. In Morse homology one builds a chain complex as follows. Critical points of a Morse function are used as a basis of a vector space which is graded by the Morse index and a boundary operator is constructed by counting gradient flow lines. As an application of Morse homology one obtains the Morse inequalities, namely a lower bound on the number of critical points of a Morse function in terms of topological data. Periodic orbits can be detected variationally as critical points of the action functional of classical mechanics. However, both the index as well as the coindex at critical points are infinite. Nevertheless, Floer managed to construct a semi-infinite dimensional Morse homology for this action functional, nowadays referred to as Floer homology. Although stable and unstable manifolds in this theory are infinite dimensional, their intersection consists of finite dimensional moduli spaces of gradient flow lines. In Floer's case the gradient flow equation is a perturbed Cauchy Riemann equation which is local, namely a PDE. An example of a nonlocal Floer homology is Rabinowitz Floer homology. Where in classical Floer homology one searches for periodic orbits of fixed period but arbitrary energy, in Rabinowitz Floer homology one is interested in periodic orbits of fixed energy but arbitrary period. While the critical point equation in Rabinowitz Floer homology is still local, namely an ODE, its gradient flow equation is not a PDE anymore. Neumann showed 1868 how Weber's force law can be derived variationally by considering retarded functionals. Such functionals give rise to problems where not only the gradient flow equation but as well the critical point equation are not local anymore and the construction of a Floer homology for such functionals is a topic at the **forefront of modern research**. References [CF09, AFS18, AFS19b, FW19, Fra19].
There will be five lectures:

1. Morse and Floer homology
2. Rabinowitz Floer homology
3. The iterated graph construction and Hamiltonian delay equations
4. Neumann functionals and Weber's Hamiltonian
5. Deformations of symplectic structures and nonlocal Floer homologies

3.1.4 C4 - Salomão - Celestial mechanics and systolic geometry

There will be three lectures:

1. Systolic inequalities for geodesics flows; the special case of the two-sphere; the Calabi-Croke example; pinching conditions; surfaces of revolution; Zermelo navigation data.

2. Systolic inequalities for Reeb flows. The case of the tight three-sphere; a conjecture of Viterbo; unboundedness of systolic ratio for general Reeb flows.

3. Some systolic inequalities for the circular planar restricted three body problem and the rotating Kepler problem.

3.1.5 C5 - Suhr - Introduction to Lorentzian geometry

The course will give an introduction to Lorentzian geometry, the mathematical language of relativistic physics, focusing especially on their geodesics. The theory of geodesics in Lorentzian manifolds is quite different from the usual one for geodesics in Riemannian geometry. First there are the observations that the energy does not have extrema and that only certain geodesics are local extrema of the length. This severely restrict any Morse theoretic approach to study the geodesics in Lorentzian geometry. Second there are global aspects of Lorentzian geometry, i.e. causality theory, which are needed to ensure existence of connecting or closed geodesics. I will illustrate all phenomena by examples. The talks will cover the following subjects:

1. Introduction to Lorentzian geometry

2. Morse theory for non-spacelike geodesics

3. Existence results for geodesics in Lorentzian geometry

3.1.6 T1 - Ragazzo - Viscoelastic tides and dissipative forces in celestial mechanics

In this talk we present equations of motion for a system of linear visco-elastic bodies interacting under gravity. The equations are obtained within a finite dimension Lagrangian framework with dissipation function. The main dynamical consequences of the dissipative tidal forces will be discussed.

3.1.7 Poster Session

João Paulo Chaib *"How to defend the validity of Newton's 3rd law in electrodynamics with four experimental results"* Poster

A.-M. Ampère (1775-1836) reported in the last part of his main work "Theory of electrodynamic phenomena, uniquely deduced from experience" two sets of experiments devised by himself in order to defend that Newton's 3rd law must be taken into account in the interaction between electrodynamic elements.

In the first set of experiments Ampère has shown how to deduce that a rigid body cannot change its movement state by electrodynamic interactions that originate from himself. This result is independent of the expression adopted for the electrodynamic interaction, or if the action is mediated or a distance.

In the second set of experiments Ampère has shown that there exist some experimental results that lead the followers of the "Elementary torque" to an interpretation that violates the first conclusion. In this work we added one new experiment in order to turn the defense more robust by using current-current interactions exclusively. We reduce the defense of the action-reaction principle (along the line joining the two interacting elements) to three experimental variations from the same set plus the work of Assis-Bueno about Ampère's bridge.

3.2 Daily schedule

17-21 February 2020 www.ime.unicamp.br/∼joa/WED.html

Mo	12:00-13:00		registration (IMECC auditório)
	13:00-14:00	C1	Cieliebak
	14:15-15:15	C2	Assis
	15:15-15:45		coffee break 1
	15:45-16:45	C3	Frauenfelder
	17:00-18:00	C4	Salomão
	18:00-19:00		Reception
Tu	09:00-10:00	C4	Salomão
	10:15-11:15	C2	Assis
	11:15-12:00		coffee break 2 and POSTER PRESENTATION
	12:00-13:00	C3	Frauenfelder
	13:00-15:00		lunch
	15:00-16:00	C1	Cieliebak
	16:00-16:30		coffee break 3
	16:30-17:30	C5	Suhr
We	**08:15**-09:15	C1	Cieliebak
	09:30-10:30	C2	Assis
	10:30-11:00		break (coffee, juice, and small food → prepare ride to Holambra)
	11:00-12:00	C3	Frauenfelder
			Advanced school excursion to Hol(land)ambra
	12:00		**bus leaves** from IMECC to Holambra
	13:00-14:00		lunch Clube Fazenda (everyone pays individually)
	14:00-18:00		Town of Holambra (map): Dutch Windmill, Parque van Gogh, much more to explore, nice area near Martin Holandesa
	18:00		**bus leaves** from lunch place Clube Fazenda to Rancho
	19:00-21:30		Advanced School Dinner at Rancho de cachaça
	21:30		**bus leaves** from Rancho back to UNICAMP
Th	09:00-10:00	C1	Cieliebak
	10:15-11:15	C2	Assis
	11:15-12:00		coffee break 4
	12:00-13:00	C3	Frauenfelder
	13:00-15:00		lunch
	15:00-16:00	C4	Salomão
	16:00-16:30		coffee break 5
	16:30-17:30	C5	Suhr

Fr	09:00-10:00	C1	Cieliebak
	10:15-11:15	C2	Assis
	11:15-12:00		coffee break 6
	12:00-13:00	C3	Frauenfelder
	13:00-15:00		lunch
	15:00-16:00	T1	Grotta Ragazzo
	16:00-16:30		coffee break 7
	16:30-17:30	C5	Suhr

References

[AC15] André Koch Torres Assis and J. C. M. C. Chaib. *Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience*. Apeiron, Montreal, 2015. Available at www.ifi.unicamp.br/∼assis.

[AFS18] Peter Albers, Urs Frauenfelder, and Felix Schlenk. What might a Hamiltonian delay equation be? *arXiv e-prints*, February 2018. arXiv:1802.07453.

[AFS19a] Peter Albers, Urs Frauenfelder, and Felix Schlenk. A compactness result for non-local unregularized gradient flow lines. *J. Fixed Point Theory Appl.*, 21(1):34–61, 2019. arXiv:1802.07445.

[AFS19b] Peter Albers, Urs Frauenfelder, and Felix Schlenk. An iterated graph construction and periodic orbits of Hamiltonian delay equations. *J. Differential Equations*, 266(5):2466–2492, 2019. arXiv:1802.07449.

[AH07] André Koch Torres Assis and J. A. Hernandes. *The Electric Force of a Current: Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents*. Apeiron, Montreal, 2007. Available at www.ifi.unicamp.br/∼assis.

[AS19] Alberto Abbondandolo and Felix Schlenk. Floer Homologies, with Applications. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 121(3):155–238, Sep 2019. arXiv: 1709.00297.

[Ass94] André Koch Torres Assis. *Weber's Electrodynamics*. Springer, Dordrecht, 1994.

[Ass14] André Koch Torres Assis. *Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force*. Apeiron, Montreal, 2014. Available at www.ifi.unicamp.br/∼assis.

[CF09] Kai Cieliebak and Urs Adrian Frauenfelder. A Floer homology for exact contact embeddings. *Pacific J. Math.*, 239(2):251–316, 2009.

[CFvK17] Kai Cieliebak, Urs Frauenfelder, and Otto van Koert. Periodic orbits in the restricted three-body problem and Arnold's J^+-invariant. *Regul. Chaotic Dyn.*, 22(4):408–434, 2017.

[CFZ19] Kai Cieliebak, Urs Frauenfelder, and Lei Zhao. J^+-invariants for planar two-center Stark-Zeeman systems. *arXiv e-prints*, page arXiv:1910.04570, Oct 2019.

[Fra19] Urs Frauenfelder. State dependent Hamiltonian delay equations and Neumann one-forms. *arXiv e-prints*, Apr 2019. arXiv:1904.01808.

[FvK18] Urs Frauenfelder and Otto van Koert. *The restricted three-body problem and holomorphic curves*. Pathways in Mathematics. Birkhäuser/Springer, Cham, 2018.

[FW19] Urs Frauenfelder and Joa Weber. The fine structure of Weber's hydrogen atom: Bohr–Sommerfeld approach. *Zeitschrift für angewandte Mathematik und Physik*, 70(4):105–116, 2019. SharedIt.

[Gei16] Hansjörg Geiges. *The geometry of celestial mechanics*, volume 83 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 2016.

[HWZ17] Helmut Hofer, Krzysztof Wysocki, and Eduard Zehnder. Polyfold and Fredholm Theory. 714 pages, *ArXiv e-prints*, page arXiv:1707.08941, July 2017.

[Neu68] Carl Neumann. *Die Principien der Elektrodynamik: eine mathematische Untersuchung*. Tübinger Universitätsschriften a. d. J. 1868. H. Laupp, 1868. Reprinted In Mathematischen Annalen, Vol. 17, pp. 400 - 434 (1880).

[WAW11] Gudrun Wolfschmidt, André Koch Torres Assis, and Karl Heinrich Wiederkehr. *Weber's Planetary Model of the Atom*. Tredition, Hamburg, 2011.

[WAW18] Gudrun Wolfschmidt, André Koch Torres Assis, and Karl Heinrich Wiederkehr. *Weber's Planeten-Modell des Atoms*. Apeiron, Montreal, 2018. Available at www.ifi.unicamp.br/∼assis.

[Web46] Wilhelm Weber. Elektrodynamische Maassbestimmungen über ein allgemeines Grundgesetz der elektrischen Wirkung. *Abhandlungen bei Begründung der Königl. Sächs. Gesellschaft der Wissenschaften am Tage der zweihundertjährigen Geburtstagfeier Leibnizen's*, 1846. Reprinted in Wilhelm Weber's Werke (Springer, Berlin, 1893), Vol. 3, pp. 25 - 214.

[Web17] Joa Weber. *Topological methods in the quest for periodic orbits*. Publicações Matemáticas do IMPA. [IMPA Mathematical Publications]. Instituto Nacional de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, 2017. 31º Colóquio Brasileiro de Matemática. vii+248 pp. ISBN: 978-85-244-0439-9. Access book. Version on arXiv:1802.06435. An extended version to appear in the EMS Lecture Series of Mathematics.

[Web18] Joa Weber. *M-Polyfold Theory*. Open Math Notes, OMN: 20180.110774. American Mathematical Society (AMS), 2018, ii+107 pp.